Problem Set 4: Recursion and Caesar Cipher

Pset Buddy

You do not have a buddy assigned for this pset.

Introduction

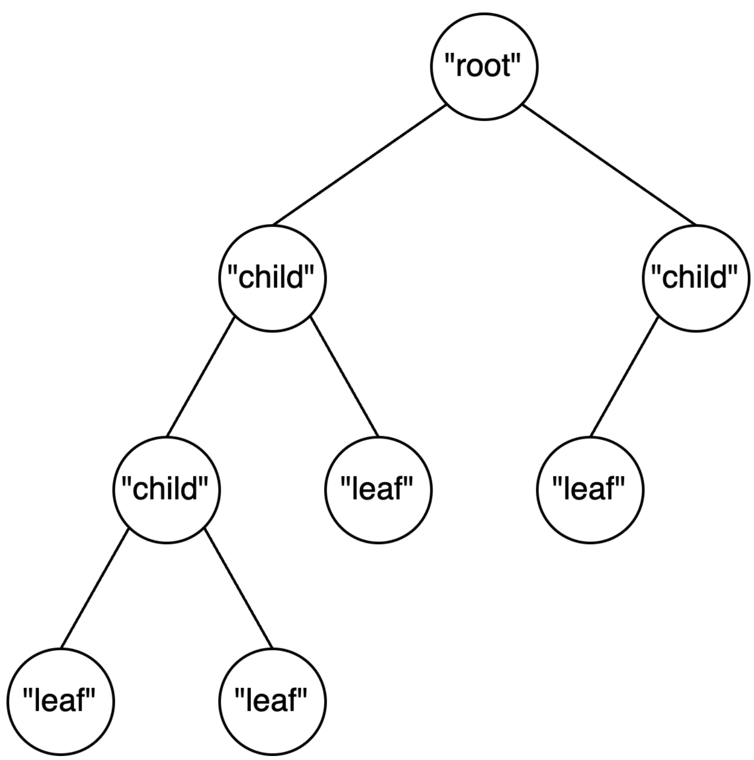
This problem set is split into two topics: the first focuses on recursion and trees (Part A), while the second looks at classes and cryptography (Parts B and C). The two topics are not dependent on one another, so feel free to work on them in parallel, or out-of-order. Please carefully read through the instruction for each.

Do not rename the files we provide you with, change any of the provided helper functions, change function/method names, or delete provided docstrings. You will need to keep **words.txt**, **story.txt** and **pads.txt** in the same folder in which you store your **.py** files.

Finally, please consult the Style Guide, as we will be taking point deductions for violations (e.g. non-descriptive variable names and uncommented code). For this pset style guide numbers **6, 7 and 8** will be highly relevant so make sure you go over those before starting the pset and again before you hand it in!

1) Part A: Recursive Operations on Trees

A tree is a hierarchical data structure composed of linked nodes. The highest node is called the *root*, which has *branches* that link it to other nodes, which are themselves roots of their respective *subtrees*. A simple tree is shown below:

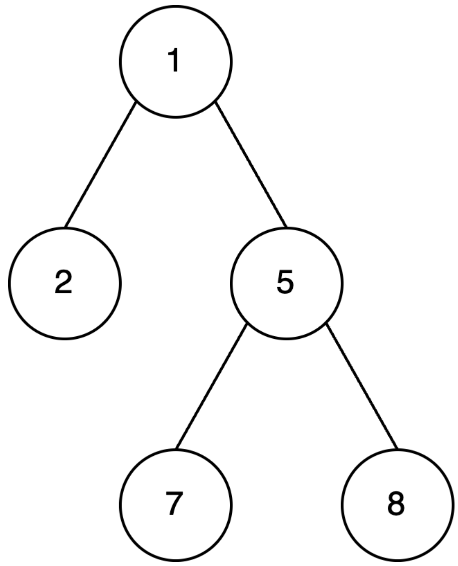


We can make a few observations:

- Each node can hold data: in this example they are holding a string with their node type.
- Data is hierarchical: each node has a **parent** (except the root), each non-leaf node has **1 or more children**, and each leaf node has **0 children**. We will be using this nomenclature in the rest of the problem set.
- Trees are inherently recursive: a root's children nodes are "roots" of other smaller trees (called **subtrees**).

1.1) Data Representation Practice

In this problem set, we will be using a provided `Node` object in **tree.py** to represent trees.



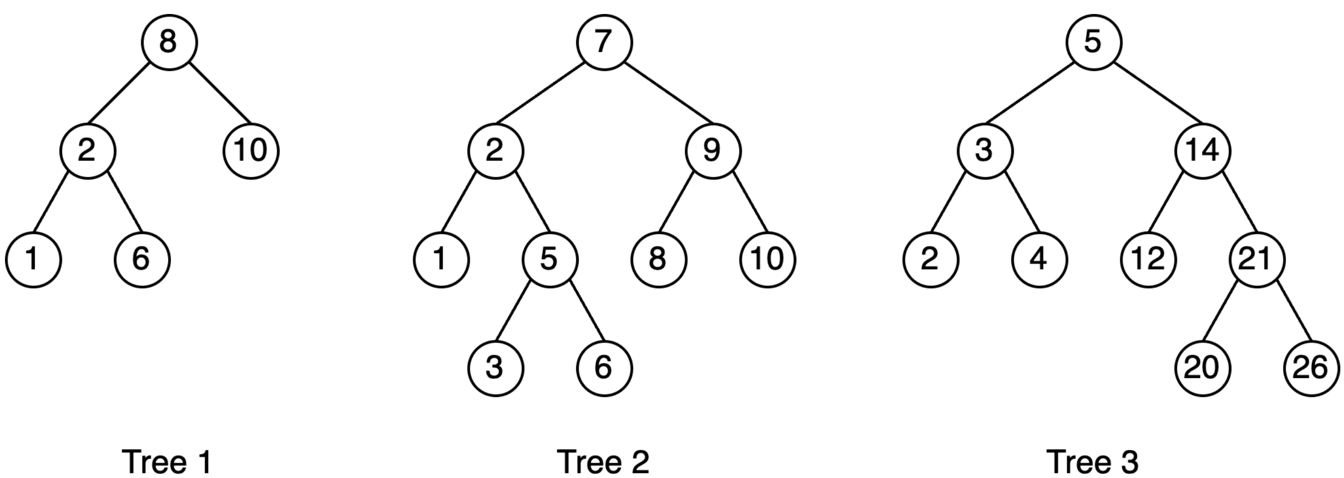
The simple tree above can be initialized with the `Node` object as follows:

```
example_tree = Node(1, Node(2), Node(5, Node(7), Node(8)))
```

A brief explanation of the Node class is below:

- You can initialize a node with the following `Node(value, left_child, right_child)`. `value` holds the value held in the node, `left_child` optionally holds the Node constructing the left subtree, `right_child` does the same for the right subtree. If there is not a subtree, either do not input that parameter *or* pass in `None`.
- You can get the `Node` object holding the left or right subtrees with `get_left_child()` or `get_right_child()` respectively. If there is no child, this function returns `None`.
- You can get the value held by a `Node` with `get_value()`.

We will practice initializing trees in this part. For the trees shown below, create objects accurately representing the data. Put them into the variables at the top of **ps4a.py**, named `tree1`, `tree2`, and `tree3`.



1.1.1) Testing

You can test out your representation so far by running **test_ps4a_student.py**. Make sure it's in the same folder as your problem set. When you correctly initialize the three tree variables, the test named `test_data_representation` in **test_ps4a_student.py** should pass.

1.2) Determining The Height of a Tree

The height of a tree is the number of edges between the root and the furthest leaf. For example, in the trees you initialized above tree 1 has a height of 2 and trees 2 and 3 have a height of 3. Write a recursive function, `find_tree_height`, that determines the depth of a tree. **This function must be recursive; non-recursive implementations will receive a zero.**

Hint. The following approach may prove to be helpful:

- Given an input tree, T:
  - **Base case**. If T is a leaf, it has a height of 0.
  - **Recursive case**. Recursively find the height of T's left and right subtrees and take the maximum of the two. Add 1 to the maximum height and return that value.

You should test your function using the variables from the previous part. For example:

```
find_tree_height(tree1) # should be 2
find_tree_height(tree2) # should be 3
find_tree_height(tree3) # should be 3
```

1.2.1) Testing

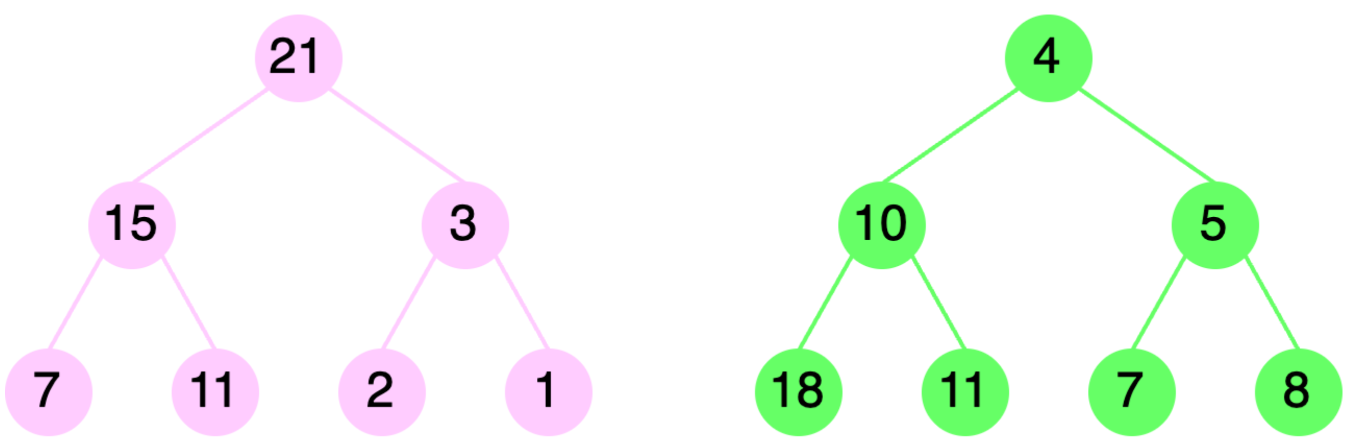
Now, your code should also pass the tests `test_tree_height` and `tree_tree_height_additional` in **test_ps4a_student.py**.

1.3) Heaps

A special type of trees are **heaps**. There are two types of heaps: **max heaps** and **min heaps**.

In a **max heap**, for each node N, N is the **maximum** value in the tree rooted at N. This means that all of the elements stored in N's left and right subtrees are *less than* than the value stored in N.

Conversely, in a **min heap**, N is the **minimum** value in the tree rooted at N, so all of the elements stored in N's left and right subtrees are *greater than* the value stored in N.



Max Heap

Min Heap

Write the function `is_heap` to quickly determine if a tree is a max or min heap, depending on the `compare_func` parameter. The `compare_func` is a function that takes in two arguments, `child_value` and `parent_value`. For max heaps, this function will return `True` if `child_value < parent_value` and `False` otherwise. For min heaps, the function will return `True` if `child_value > parent_value` and `False` otherwise. **Conceptually, this allows you to write one function that can determine both max and min heaps based on a parameter, instead of writing two separate methods with very similar code.** Below, we have provided you with working implementations of `compare_func` for a max and min heap, respectively.

```python
# max heap comparator
def compare_func(child_value, parent_value):
    if child_value < parent_value:
        return True
    return False

# min heap comparator
def compare_func(child_value, parent_value):
    if child value > parent_value:
        return True
    return False
```

Hint. The following approach may prove to be helpful:

- Given an input tree, T:
  - **Base case.** If T is a leaf then it is a heap (for both max and min heaps)
  - **Recursive case.** For a node T, recursively check if T's right and left subtrees are also heaps and that T is the max or min element (using the comparator function `compare_func` passed to `is_heap`) of its subtree. If all the previous conditions are true, then the tree rooted at T is a heap. Otherwise, it is not.

1.3.1) Testing

You should now pass all of the tests in **test_ps4a_student.py**.

2) Part B: Encryption with One Time Pads

2.1) Introduction

In this problem we will implement a simple encryption technique that when implemented correctly cannot be broken.

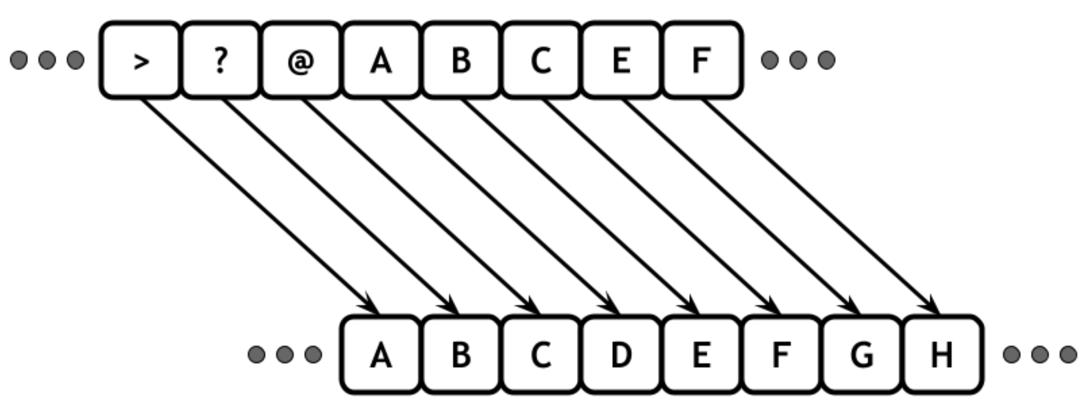
Here is some important vocabulary we will use going forward:

- **encryption**. the process of obscuring or encoding messages to make them unreadable.
- **decryption**. the process of converting encrypted messages back to their original, readable form.
- **plaintext**. the original, readable message.
- **ciphertext**. the encrypted message. A ciphertext still contains all of the original message information, even though it looks like gibberish.

2.1.1) How a One Time Pad Works

The idea of a one time pad is to "shift" each character in your plaintext message by a random amount. This results in a ciphertext that cannot be decrypted without the list of random amounts your letters were shifted by, called the **pad**.

In this pset we want our messages to be able to include letters, numbers, spaces and other special characters. Shifting a letter makes sense (for example `a` shifted by 3 would be `d`), but what would a space shifted by 3 be? Luckily, all characters are represented on our computers as numbers already! You can view the mappings of basic characters to numbers in this ASCII table. In it we can see that a space is represented by the numeric value `32`, so shifting it by 3 results in `35` or the character `#`. An example of shifting letters by 3 is pictured below.



We need to be careful to properly handle the case where the shift wraps around to the start. We're only interested in characters with ASCII values from `32` to `126`. So the last character `~` with the value `126` shifted by 1 would be a space (value `32`) and `~` shifted by 5 would be a `$` (value `36`). Some more examples are found in the following table:

Character (& ASCII Value)	Shift value	Shifted Character (& ASCII Value)
"A" (65)	5	"F" (70)
"a" (97)	10	"k" (107)
" " (SPACE, 32)	-1	"~" (126)
"-" (45)	8	"5" (53)
"}" (125)	4	""" (QUOTE, 34)
"k" (107)	-3	"h" (104)
"Y" (89)	-12	"M" (77)
"Y" (89)	-107	"M" (77)

Now that we know how to shift characters, to use a one time pad we simply have to shift each character in our message by a random amount specified by the pad. For example, if we have the message `hello` (ASCII values `[104, 101, 108, 108, 111]`) and the pad `[3, 0, 10, 11, 4]`, we would do compute `[104+3, 101+0, 108+10, 108+11, 111+4]`, which equals `[107, 101, 118, 119, 115]`. Converting back from values to characters, our ciphertext would be `kevws`. More examples are in the chart below:

Plaintext	One Time Pad	Process	Ciphertext
`Aaa`	1,2,3	65+1=66, 97+2=99, 97+3=100	`Bcd`
`xyz`	2,10,12	120+2=122, 121+10=36, 122+12=39	`z$'`
`Hello!`	5,10,2,3,0,2	72+5=77, 101+10=111, 108+2=110, 108+3=111, 111+0=111, 33+2=35	`Monoo#`
`Monoo#`	-5,-10,-2,-3,0,-2	77-5=72, 111-10=101, 110-2=108, 111-3=108, 111+0=111, 35-2=33	`Hello!`

Now that we know how one time pads work we can start implementing them!

2.1.2) Using Classes and Inheritance

This is your first experience creating your own classes! Get excited! They are a powerful idea that you will use throughout the rest of your programming career. If you need a refresher on how classes and inheritance work, refer to the lecture and recitation notes.

For this problem set, we will use a parent class called `Message`, which has two child classes: `PlaintextMessage` and `EncryptedMessage`.

- `Message` contains methods that both plaintext and encrypted messages will need to use. For example, a method to get the text of the message. The child classes will inherit these shared methods from their parent.
- `PlaintextMessage` contains methods that are specific to a plaintext message, such as a method for generating a one time pad or encrypting a message.
- `EncryptedMessage` contains methods that are specific to a ciphertext, such as a method to decrypt a message given a one time pad.

2.2) `Message`

We have provided skeleton code in the `Message` class for the following functions. **Your task is to fill in the methods of the `Message` class** found in **ps4b.py** according to the specifications in the docstrings. Please see the docstring comment of each function for more information about the function's specification.

- `__init__(self, input_text)`
- `get_text(self)`
- `shift_char(self, char, shift)`. This should return a string containing `char` shifted by `shift` according to the method described above. Some hints to keep in mind:
  - `ord(char)` returns the ASCII value of a string `char` which contains a single character
  - `chr(ascii_num)` returns a string with the single character specified by `ascii_num`
  - We are only interested in the **95** ASCII characters from ASCII values `32` to `126`
  - The modulo operator `%` which returns the remainder of division is helpful for "wrapping around".
- `apply_pad(self, pad)`. This should return a string containing the ciphertext of `self.message_text` after `pad` is applied

We have also implemented a `__repr__` function so that when you print out your `Message` objects, the console displays a nice human readable result. **Please do not change it.**

2.2.1) Testing

You can test out your code so far by running **test_ps4bc_student.py**. Make sure it's in the same folder as your problem set. At this point, your code should be able to pass all the tests beginning with `test_message`, but not the ones beginning with `test_plaintext_message` or `test_encrypted_message`. We will implement code for those in the next section.

2.3) `PlaintextMessage`

Again, your task is to **fill in the methods of the `PlaintextMessage` class** found in **ps4b.py** according to the specifications in the docstrings.

- `__init__(self, input_text, pad=None)`
  - You should use the parent class constructor (using `super()`) in this method to make your code more concise. Take a look at Style Guide #7 if you are confused.
  - The syntax `pad=None` indicates an optional argument that can be omitted and the specified default value of `None` is passed in instead.
    - For example, `PlaintextMessage('test')` and `PlaintextMessage('test', [0,15,3,9])` are both valid constructors but the former should generate a random pad and the latter should use the specified pad.
  - You should save a **copy** of `pad` as an attribute and not `pad` directly to protect it from being mutated.
- `generate_pad(self)`
  - Note: our `shift_char` should work for any arbitrary integer but for simplicity we'll only generate pads with integers in the range `[0, 110)`
  - Hint: `random.randint(a, b)` returns a random integer N such that $a \leq N \leq b$.
- `get_pad(self)`
  - This should return a **copy** of `self.pad` to prevent someone from mutating the original list.
- `get_ciphertext(self)`
- `change_pad(self, new_pad)`
  - Make sure `self.get_ciphertext` uses the new pad!

We have also implemented a `__repr__` function so that when you print out your `PlaintextMessage` objects it returns a nice human readable result. **Please do not change it.**

2.3.1) Testing

You can test your new class by running **test_ps4bc.py**. You should now be able to pass all the tests starting with `test_message` and `test_plaintext`.

2.4) `EncryptedMessage`

Given an encrypted message, if you know the one time pad used to encode the message, decoding it is trivial. That's because if we shifted a character by x to encrypt it then to decrypt it we simply shift it back by $-x$! Therefore to decrypt a message that was shifted with the onetime pad `[i, j, k, ...]` we simply apply the pad `[-i, -j, -k, ...]`. So if `$ip!` is the encrypted message, and `[5, 1, 7, 2]` was the pad used to encrypt the message, then `[-5, -1, -7, -2]` decodes the encrypted message and gives you the original plaintext message.

Ciphertext	One Time Pad	Process	Plaintext
`$ip!`	5,1,7,2	36-5=126, 105-1=104, 112-7=105, 33-2=126	`~hi~`

We now will implement decryption in the `EncryptedMessage` class.

Fill in the following methods of the `EncryptedMessage` class found in **ps4b.py** according to the specifications in the docstrings.

- `__init__(input_text)`
  - As with `PlaintextMessage`, use the parent class constructor (`super()`) to make your code more concise. Take a look at Style Guide #7 if you are confused.
- `decrypt_message(self, pad)`
  - This function should be very short. Using the above explanation of how to decrypt a one-time pad, try to use a function you already wrote in the `Message` class.

2.4.1) Testing

You can test your new class by running **test_ps4bc_student.py**. You should now be able to pass all the tests in that file, **except** for `test_try_pads`.

3) Part C: Using Your Classes

Now that we have created our classes in **ps4b.py** we will learn how to use them in **ps4c.py**! At the top of **ps4c.py** you can see the line:

```
import ps4b # Importing your work from Part B
```

This imports your code from the **ps4b.py** file if it is in the same folder. To use a class created in Part B you can initialize it like this: `my_message = ps4b.Message("My Message!")`.

There are a few helper functions we have implemented for you: `load_words`, `is_word`, and `get_story_string`. These will be helpful for implementing `decrypt_message_try_pads` and `decode_story`. You don't need to understand exactly how they work, but you should read their associated docstrings to understand what they do and how to use them.

3.1) Decoding Ciphertexts

One time pads are secure so we can't find a plaintext message without the pad used to encrypt it. However, if we have a ciphertext and a list of the pads which might have been used to encrypt it, we can find which pad actually encrypted it and what the plaintext message is.

To do this programmatically we would try decrypting the ciphertext with each pad in the list and count the number of English words in the output. We can count the number of words by spliting the decrypted output on spaces and testing if each is a valid English word. Then we can assume that pad that produces a plaintext message with the most valid words was the pad used to encrypt the message. Additionally, in the event of ties, we want to return the **last pad** that results in the maximum number of valid English words.

Fill in `decrypt_message_try_pads(self, pads)` using the approach described above.

You may find the helper functions `is_word(wordlist, word)` and `load_words`, and the string method `split` useful. *Note that `is_word` will ignore punctuation and other special characters when considering whether or not a word is valid.*

3.1.1) Testing

You should now be able to pass all the tests in **test_ps4bc_student.py**.

Please note that in the student tester your `decrypt_message_try_pads` will rely on your work from **ps4b.py**. However, the staff tester will have one test where it will use our reference staff implementation of **ps4b.py**. Therefore **you should make sure that you are using your getters and setters** rather than directly accessing class attributes from `decrypt_message_try_pads`!

3.1.2) Decoding a Story

Bob is trying to share a story with Alice without us being able to learn the story. Fortunately for us we overheard Bob when he was sharing all of his one time pads with Alice; we just don't know which pad he used for his story.

Implement `decode_story` to find what Bob's story was.

- use `get_story_string` to get the ciphertext of Bob's story and `get_story_pads` to get a list of Bob's one time pads.
- Use your `decrypt_message_try_pads` function to find what Bob's message to Alice was.

Check the output of `decode_story` by uncommenting the code at the bottom of **ps4c.py** and running **ps4c.py**. This function is **not tested** in **test_ps4bc_student.py**, but be prepared to discuss the decrypted story at your checkoff.

4) Hand-in Procedure

4.1) Time and Collaboration Info

At the start of each file, in a comment, write down the names of your collaborators. For example:

```
# Problem Set 4B
# Name: Jane Lee
# Collaborators: John Doe
```

Please estimate the number of hours you spent on the Problem Set in the question box below.

4.2) Half-way Submission

All students should submit their progress by the half-way due date (1 week before the final due date).

This submission will be worth 1 point out of the problem set grade and will not be graded for correctness. The intention is to make sure that you are making steady progress on the problem set as opposed to working on it in the final days before the due date.

You may upload new versions of **ps4a.py** until Nov 09 at 09:00PM. You cannot use extensions or late days on this submission.

Please refresh the page before submitting a new file. If you do not, your latest submission won't be updated.

Select File No file selected

Submit

You have infinitely many submissions remaining.

4.3) Final Submission

Be sure to run the student testers `test_ps4a_student.py`, and `test_ps4bc_student.py` and make sure all the tests pass. However, the student tester contains only a subset of the tests that will be run to determine the problem set grade. Passing all of the provided test cases does not guarantee full credit on the pset.

You may upload new versions of each file until Nov 16 at 09:00PM, but anything uploaded after that time will be counted towards your late days, if you have any remaining. If you have no remaining late days, you will receive no credit for a late submission.

When you upload a new file with the same name, your old one will be overwritten.

4.3.1) Part A

Select File No file selected

Submit

You have infinitely many submissions remaining.

4.3.2) Part B

Select File No file selected

Submit

You have infinitely many submissions remaining.

4.3.3) Part C

Select File No file selected

Submit

You have infinitely many submissions remaining.